TRANSIENT REGIMES OF THERMOELECTRIC COOLING AND HEATING UNITS

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The author examines the differential equations characterizing the transient regimes of semiconductor cooling and heating units. The voltage of the source of electric power supply is taken as one of the independent parameters.

In the theory of thermoelectric cooling and heating the design quantities are usually determined by the temperature conditions and the current flowing through the theromelements. The advantage of these formulas is in their simplicity. For example, in the known equations for cooling capacity (1) and power input (2), the physical meaning of the energetic processes taking place in the thermocouple (Peltier and Seebeck effects, Joule and Fourier heating) is clearly seen [1]:

$$Q_0 = eT_0I - \frac{1}{2}I^2R - k(T - T_0) \tag{1}$$

$$W' = I^2R + eI(T - T_0). \tag{2}$$

However, thermoelectric units, like most units that consume electric power, are connected to a supply voltage whose value does not depend on time. In this case, the electric current is not always a constant parameter. In units that work under periodically varying temperature conditions or in nonstationary regimes, changes in junction temperature affect the Seebeck emf and change the current in the circuit. In the steady-state regime at constant temperatures, voltage and current are equivalent parameters.

If we eliminate the current from (1) and (2) by making the substitution

$$I = [U - e(T - T_0)]/R, \tag{3}$$

we obtain the following equations for the cooling capacity and power input:

$$Q_0 = \frac{UeT}{R} - \frac{1}{2}\frac{e^2}{R}(T^2 - T_0^2) - \frac{1}{2}\frac{U^2}{R} - k(T - T_0), \tag{4}$$

$$W' = \frac{U}{R}[U - e(T - T_0)]. \tag{5}$$

The advantage of these formulas is that they are expressed in terms of a constant parameter – the supply voltage. Therefore, transient regimes may conveniently be analyzed using Eqs. (4) and (5).

Transient processes in thermoelements were first examined in [2]. The results obtained had great theoretical significance, but the derived formulas were expressed in terms of current, which made them difficult to apply. Similar shortcomings affect the results of [3, 4].

This note is concerned with transient regimes in semiconductor thermoelectric units designed for cooling and heating various bodies. No account is taken of the rapid supercooling or superheating of the junctions due to the fact that the Peltier heat is absorbed and released directly at the junctions whereas the Joule and Fourier heating is realized in the body of the thermocouple [2]. Moreover, the following assumptions are made.

The thermoelectric parameters of semiconductor materials – thermal emf, electrical resistivity and conductivity – do not depend on temperature; the temperature of the surrounding medium and the heat capacity of the cooled or heated body are assumed constant; the thermal resistances between the junctions, the surrounding medium, and the body and taken into account as the time-averaged temperature differences necessary for heat transfer.

Cooling regime. Consider the cooling of a body with mass m referred to one thermocouple and specific heat c. If we neglect heat inflow through the insulation, then in time $d\tau$ the thermocouple cools the body by an amount $mcdT_0$, and the differential equation of the cooling process takes the form

$$-mcdT_0/d\tau = \frac{1}{2}\frac{e^2}{R}T_0^2 + kT_0 + \frac{UeT}{R} - \tag{6}$$

$$-\frac{1}{2}\frac{U^2}{R}-\frac{1}{2}\frac{e^2}{R}T^2-kT. \qquad (6)\ \text{(cont'd)}$$

If allowance is made for heat inflow through the insulation

$$-mcdT_0/d\tau+kF(T-T_0)=\frac{1}{2}\frac{e^2}{R}T_0^2+kT_0+ \\ +\frac{UeT}{R}-\frac{1}{2}\frac{U^2}{R}-\frac{1}{2}\frac{e^2}{R}T^2-kT. \qquad (7)$$

Here kF is the insulating value of the insulation referred to one thermocouple.

Since (7) may be reduced to the form (6) by making the substitution $k' = k + kF$, henceforth we will consider only equations of type (6). Solution of this differential equation with the initial condition $\tau = 0$, $T_0 = T_{01}$ gives a value of the time for cooling the body from temperature T_{01} to T_0:

$$\tau=\frac{mc}{kL}\ln\frac{(zT_{01}+1-L)(zT_0+1+L)}{(zT_{01}+1+L)(zT_0+1-L)}. \qquad (8)$$

Here

$$L=\sqrt{1+2zT+z^2e^{-2}(eT-U)^2}. \qquad (9)$$

The range of voltages for which the cooling time and cooling capacity of the thermocouples take positive values is given by the equation

$$L=zT_0+1. \qquad (10)$$

To obtain the minimum cooling time it is necessary to pass through the thermoelements a current corresponding to the maximum cooling capacity

$$I_{\tau_{min}}=eT_0/R. \qquad (11)$$

The temperature of the cold junction (cooled body) changes with time; therefore the current $I_{\tau_{min}}$ is not constant. However, to obtain the current (11) is is necessary to connect the thermocouple to a power supply whose voltage is constant with time and equal to

$$U_{\tau_{min}}=eT. \qquad (12)$$

In this case the minimum cooling time regime is maintained automatically.

Since the voltage $U_{\tau_{min}}$ is constant, replacing U by $U_{\tau_{min}}$ in (6) does not affect the solution. Therefore, for determining τ_{min} it is sufficient to substitute $U = eT$ in Eq. (8) or to replace L by

$$N=\sqrt{1+2zT}. \qquad (13)$$

If the thermoelectric unit is to work with maximum efficiency in the transient regime, then it is necessary to pass through the thermoelements a current

$$I_{\varepsilon_{max}}=e(T-T_0)/(M-1)R. \qquad (14)$$

In equation (14)

$$M=\sqrt{1+(T+T_0)z/2}. \qquad (15)$$

To obtain the current (14), the supply voltage must be equal to

$$U_{\varepsilon_{max}}=Me(T-T_0)/(M-1). \qquad (16)$$

Thus, under variable temperature conditions, the maximum efficiency regime can not be attained at a constant supply voltage.

The differential equation determining the cooling time in the maximum efficiency regime is found by substituting (16) in (6):

$$\frac{mc(M-1)^2(M+1)}{zkM}\,\frac{dT_0}{d\tau}-MT_0^2+T(M+1)T_0-T^2=0. \tag{17}$$

Henceforth, as is usual in thermoelectric theory, we will consider M to be constant. Then the solution of (17) with the previous initial condition gives the following value for the cooling time:

$$\tau_{\varepsilon_{\max}}=\frac{mc(M^2-1)}{zkMT}\ln\frac{(T-T_0)(MT_{01}-T)}{(T-T_{01})(MT_0-T)}. \tag{18}$$

In the transient cooling regime, the electrical power consumption in the general case can be determined as follows:

$$W=\int_0^\tau\frac{U}{R}[U-e(T-T_0)]\,d\tau. \tag{19}$$

If we consider minimum cooling time or maximum efficiency regimes, then

$$W_{\tau_{\min}}=zkT\int_0^\tau T_0 d\tau, \tag{20}$$

$$W_{\min}=\frac{zkM}{(M-1)^2}\int_0^\tau(T-T_0)^2\,d\tau. \tag{21}$$

In integrating Eqs. (19)-(21) it is necessary to take into account that $T_0 = f(\tau)$. Solving (8) and (18) for T_0, we obtain:

$$T_0=[(1+L)(zT_{01}+1-L)-(1-L)(zT_{01}+1+L)\times$$
$$\times\exp(kL\tau/mc)]\{z(zT_{01}+1+L)[\exp(kL\tau/mc)-1]+2Lz\}^{-1}; \tag{22}$$

$$T_{0\varepsilon_{\max}}=T\{(T-T_{01})[\exp(zkMT\tau/mc(M^2-1))]+MT_{01}-T\}\times$$
$$\times\{M(T-T_{01})[\exp(zkMT\tau/mc(m^2-1))]+MT_{01}-T\}^{-1}. \tag{23}$$

The equation for T_0 in the $\tau_{\min}$ regime is obtained from (22) by substituting L = N. Substituting in (19)-(21) the value of T_0 from (22)-(23) and integrating, we get

$$W=\frac{Ue}{Rz}\left[\left(L-1-zT+\frac{Uz}{e}\right)\tau+2\frac{mc}{k}\ln\frac{zT_{01}+1+L}{zT_0+1+L}\right], \tag{24}$$

$$W_{\tau_{\min}}=kT(N-1)\tau+2mcT\ln\frac{zT_{01}+1+N}{zT_0+1+N}, \tag{25}$$

$$W_{\min}=\frac{mc(M+1)}{M}\left[\frac{T(M-1)}{M}\ln\frac{MT_{01}-T}{MT_0-T}-(T_{01}-T_0)\right]. \tag{26}$$

Heating regime. In thermoelectric heating the thermocouples work in the heat pump regime. Thermoelectric heating is more economical than electric heating, since the quantity of heat Q supplied to the body is greater than the energy consumed by the external source by an amount Q_0, i.e.,

$$Q=W+Q_0. \tag{27}$$

Relationship (27) is meaningful at a body temperature $T < T_{\max}$ ($T_{\max}$ is the maximum temperature of the hot junction for a thermocouple working in the heat pump regime). If $T > T_{\max}$, then $Q_0 = 0$ and Q = W, i.e., thermoelectric heating goes over into electric. It is necessary to emphasize that, in contrast to the case of cooling, when a temperature $T_0 < T_{0\min}$ is impossible to obtain at any supply voltage, with heating the regime $T < T_{\max}$ is not only possible, but in a number of cases has practical value. If the required final temperature of the heated body exceeds $T_{\max}$, then the heating process can be conditionally divided into two stages: first, when $T < T_{\max}$, heating is thermoelectric and economizes on electrical energy; second, when $T > T_{\max}$, heating is electric and due only to the external power source. In this case the over-all heating effect involves a saving of electrical energy, even when $T > T_{\max}$.

Let us examine the heating process. In this case the variable temperature will be T, while the temperature of the surrounding medium, T_0, is constant. The heating capacity of a thermocouple, expressed in terms of the supply voltage is

$$Q = \frac{UeT_0}{R} - \frac{1}{2}\frac{e^2}{R}(T^2 - T_0^2) + \frac{1}{2}\frac{U^2}{R} - k(T - T_0). \tag{28}$$

The differential equation of heating has the form

$$mcdT/d\tau + \frac{1}{2}zkT^2 + kT - \frac{1}{2}zkT_0^2 - \frac{UeT_0}{R} - kT_0 - \frac{1}{2}\frac{U^2}{R} = 0. \tag{29}$$

Solution of this equation with initial condition $\tau = 0$, $T = T_1$ gives the time for heating the body from T_1 to T:

$$\tau = \frac{mc}{kL'}\ln\frac{(L' - zT_1 - 1)(L' + zT + 1)}{(L' - zT - 1)(L' + zT_1 + 1)}, \tag{30}$$

where

$$L' = \sqrt{1 + 2zT_0 + z^2e^{-2}(U + eT_0)^2}. \tag{31}$$

It is obvious that there is no τ_{min} regime and that with increasing voltage the heating time continuously decreases. For given temperature conditions the minimum value of the voltage supplied to the thermoelement is given by

$$L' = zT + 1. \tag{32}$$

To obtain maximum efficiency, it is necessary to vary the supply voltage in accordance with Eq. (16), taking into account that the variable temperature is T. In this case the heating equation takes the form

$$mc\frac{dT}{d\tau} = \frac{zkM}{(M - 1)^2(M + 1)}(T - T_0)(MT - T_0), \tag{33}$$

and its solution with the previous initial condition is:

$$\tau_{\varphi max} = \frac{mc(M^2 - 1)}{zkMT_0}\ln\frac{(T - T_0)(MT_1 - T_0)}{(T_1 - T_0)(MT - T_0)}. \tag{34}$$

Consumption of electrical energy in the heating regime is determined in the same way as for cooling. We arrive at the final formulas.

Energy consumption in the general case

$$W = \frac{Ue}{Rz}\left[\left(\frac{Uz}{e} + zT_0 + 1 - L'\right)\tau - 2\frac{mc}{k}\ln\frac{L' + zT + 1}{L' + zT_1 + 1}\right]. \tag{35}$$

Energy consumption in maximum efficiency regime

$$W_{min} = \frac{mc(M + 1)}{M}\left[T - T_1 - \frac{T_0(M - 1)}{M}\ln\frac{MT - T_0}{MT_1 - T_0}\right]. \tag{36}$$

Approximate formulas. In many technical calculations an accuracy of 3-5% is sufficient. Under these conditions, certain of the above relations can be simplified. In all the equations determining the cooling and heating times, the ratio of expressions in parentheses, containing only addends, differs little from unity. Therefore, with good accuracy, it is possible to base the calculations on the following formulas.

Cooling regime:

$$\tau = \frac{mc}{kL}\ln\frac{zT_{01} + 1 - L}{zT_0 + 1 - L}, \tag{37}$$

$$W = \frac{Ue}{Rz}\left(L - 1 - zT + \frac{Uz}{e}\right)\tau, \tag{38}$$

$$W_{\tau_{min}} = kT(N - 1)\tau_{min}. \tag{39}$$

Heating regime:

$$\tau = \frac{mc}{kL'} \ln \frac{L' - zT_1 - 1}{L' - zT - 1}, \qquad (40)$$

$$W = \frac{Ue}{Rz} \left(\frac{Uz}{e} + zT_0 + 1 - L' \right) \tau. \qquad (41)$$

NOTATION

e, R and k – thermal emf, electrical resistivity and reduced heat conductivity of thermocouple; $z = e^2/Rk$ – complex characteristic of semiconductor materials; T and T_0 – temperatures of hot and cold junctions; I – current flowing through thermoelement; U – supply voltage referred to one thermocouple; Q and Q_0 – heating and cooling capacities of thermocouple; W – electrical energy consumed by thermocouple; W' – power consumed by thermocouple; τ – cooling or heating time; m – mass of cooled or heated body referred to one thermocouple; c – specific heat of cooled or heated body; ε and φ – cooling and heating factors of thermoelectric unit.

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